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ENERGY STABILITY IN RECIRCULATING, ENERGY–RECOVERING LINACS

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ABSTRACT

Recirculating, energy–recovering linacs can be used as driver accelerators for high power FELs. Instabilities which arise from fluctuations of the cavity fields are investigated. Energy changes can cause beam loss on apertures, or, when coupled to M_{56}, phase oscillations. Both effects change the beam induced voltage in the cavities and can lead to unstable variations of the accelerating field. Stability analysis for small perturbations from equilibrium is performed and threshold currents are determined. Furthermore, the analytical model is extended to include amplitude and phase feedback, with the transfer function in the feedback path presently modeled as a low–pass filter. The feedback gain and bandwidth required for stability are calculated for the high power UV FEL proposed for construction at CEBAF.

1. STABILITY ANALYSIS

The interaction of the beam with the cavity fields can be described, to a very good approximation, by the following first order differential equation,

$$\frac{d\tilde{V}_c}{dt} + \frac{\omega_0}{2Q_L}(1 - i\tan\Psi)\tilde{V}_c = \frac{\omega_0 R_L}{2Q_L}(\tilde{I}_g - \tilde{I}_b) \tag{1}$$

where ω_0 is the cavity resonant frequency, Q_L is the loaded Q of the cavity and R_L is the loaded shunt impedance $R_L = (r/Q)Q_L$. The beam in the cavity is represented by a current generator. In arriving at (1) we assume that the cavity voltage, generator and beam current vary as $e^{i\omega t}$, where ω is the rf frequency, and $\tilde{V}_c$, $\tilde{I}_g$ and $\tilde{I}_b$ are the corresponding complex amplitudes (phasors) in the rotating frame of reference, varying slowly with time. In this equation I_b (absence of tilde denotes the magnitude of the corresponding quantity) is equal to the average beam current (in the limit of short bunches). Also Ψ is the tuning angle defined by $\tan\Psi = -2Q_L(\omega - \omega_0)/\omega_0$. In steady-state the generator power is given by

$$P_g = \frac{(1+\beta)}{4\beta} I_g^2 R_L \quad ,$$

where β is the cavity coupling coefficient, and can be calculated from $Q_L = Q_0/(1+\beta)$.

1A. Open Loop Analysis

As a concrete example, we take the energy-recovering driver accelerator design of the CEBAF FEL [1, 2] and follow the formalism presented in [3].

The accelerator consists of an injector, a superconducting rf linac with a two-pass recirculation transport, which accelerates the beam to 200 MeV, decelerates it for energy recovery through two passes, and transports it to a dump. Therefore, in this model, there are four beams in the linac cavities at any time (two accelerating and two decelerating).

Two effects may trigger an unstable behavior of the system: a) Beam current loss which may originate from energy offset which shifts the beam centroid off its central trajectory and leads to beam scraping on apertures. b) Phase shift which may originate from an energy offset coupled to the finite compaction factors (M_{56}) in the arcs.

Substituting the above equations into the cavity equation (1), separating real and imaginary parts, performing the linearization, and taking the Laplace transform of the equations, we obtain two algebraic equations $MA = 0$, where M is a 2×2 matrix and A is the column vector with $\hat{v}(s)$ and $\hat{\phi}(s)$ as components.

The determinant of M is then set to zero and the two roots of s are examined. The real parts of the roots will provide the damping or growth rates of perturbations. The imaginary parts of the roots will give the oscillation frequencies relative to the driving rf frequency. If both roots have zero or negative real parts, the system is stable; otherwise the system is unstable. Taking this into account, the two roots of s are

$$s = \left(\frac{\omega_0}{2Q_L}\right)\left\{-1 + \frac{I_0 R_L}{2}(h_2 S + b_2 C) \pm \sqrt{\left[\frac{I_0 R_L}{2}(h_2 S + b_2 C)\right]^2 - \Lambda}\right\}$$

where

$$\Lambda = I_0 R_L (h_2 C - b_2 S) \tan \Psi + \tan^2 \Psi$$

is a coupling term arising from the non-zero tuning angle Ψ, and S and C are defined as

$$\begin{aligned} S &= \sum_{i=1}^{2} [\sin(\Psi_i - \Psi_3) + \sin(\Psi_i - \Psi_4)] \\ C &= \sum_{i=1}^{2} [\cos(\Psi_i - \Psi_3) + \cos(\Psi_i - \Psi_4)] \end{aligned}$$

and Ψ_i , i=1,2,3,4 are the steady–state phases of the beam for passes 1,2,3,4 with respect to the cavity voltage.

In the absence of coupling ($\Lambda = 0$) and $(h_2 S + b_2 C) \leq 0$ the system is stable for all values of the beam current. For $(h_2 S + b_2 C) > 0$ however, the system becomes unstable for currents above a threshold current I_{th} given by

$$I_{\text{th}} = \frac{1}{R_L (h_2 S + b_2 C)} \quad .$$

In this case the growth rate of the instability increases linearly with the beam current. Coupling, in this parameter regime, can manifest itself as a frequency shift, and the system remains unstable.

For $(h_2S + b_2C) \leq 0$, if the coupling term is strong enough it can make the system unstable. The growth rate of this instability however, is slow and approaches asymptotically a constant value as the beam current increases.

For the CEBAF FEL design parameters, $(h_2S + b_2C) > 0$, and the system is unstable at $I_0 = 5$ mA. The instability threshold is 300 μA and the growth rate of the instability is $s \simeq 3$ kHz at $I_0 = 5$ mA. The threshold current for the longitudinal instability alone ($b_2 = 0$) is 400 μA, and for the beam loss instability alone ($h_2 = 0$) is 1.4 mA. Therefore, when both instabilities are present the threshold is dominated by the longitudinal one, for 10^{-3} losses produced by 1 mm offset.

1B. Analysis with Feedback

In the presence of feedback, the generator current $\tilde{I}_g$ is no longer constant, but it assumes the form

$$\tilde{I}_g = [I_{g0} + \Delta I_g(t)]e^{i[\Psi_{g0} + \Delta\Psi_g(t)]}$$

where $\Delta I_g(t)$ is the additional signal providing amplitude feedback, and $\Delta\Psi_g(t)$ is the additional signal providing phase feedback [4]. The transfer function in the feedback path is presently modeled as a low–pass filter with gain G and roll–off frequency $(2\pi T_1)^{-1}$. Therefore the Laplace transforms of ΔI_g and $\Delta\Psi_g$ are

$$\frac{\Delta I_g(s)}{I_{g0}} = -\frac{G}{1 + sT_1}\frac{\hat{v}(s)}{V_{c0}} \quad ,$$

and

$$\Delta\Psi_g(s) = -\frac{G}{1 + sT_1}\hat{\phi}(s) \quad ,$$

where $\hat{v}(s)$ and $\hat{\phi}(s)$ are the errors in the amplitude and phase of the cavity field.

The analysis is similar to the open loop case, only now DetM=0 is a quartic polynomial in s. We solved for the roots of s and next we summarize the results for the CEBAF FEL design parameters.

With both longitudinal and beam loss effects present, $(h_2S + b_2C) > 0$, a gain of 26 with roll–off frequency greater or equal to 3 kHz is sufficient to bring the system to the stability boundary, for small perturbations around the equilibrium. For the scraping instability alone, the required gain is 7 with 1 kHz bandwidth, while for the longitudinal instability alone the required gain is 13 with approximately 2 kHz bandwidth.

2. CONCLUSIONS

Stability analysis for small perturbations from equilibrium has been performed for a recirculating linac FEL driver accelerator with energy recovery. The feedback characteristics required to stabilize the system have been determined. For the CEBAF FEL design parameters, modest gains at reasonable frequencies (well within the range of the CEBAF rf control system) are required to stabilize the system.

Future directions include accurate modeling of the instabilities by direct numerical integration of the systems equations. This approach will allow incorporating into the model nonlinearities from the saturation of the klystron and start up transients.

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